**Plasma Dynamics**

Now let’s suppose that we have a multi-component fluid. How do things change? Well in this case, I think we have to relax the local equilibrium hypothesis, since the two components may have different velocities at the same point in space – which would be a definite non-equilibrium situation. So we’ll consider s = s1(ε1,**℘**1,n1,ψ) + s2(ε2,**℘**2,n2,ψ). Note we’re assuming the two species of particles (positive ions, negative ions) are distinguishable, and so it is legitimate to say the total entropy is the sum of the individual ones. I suppose one could also formulate the problem from the point of view that it’s just a single two-component fluid/plasma? So would then have a single energy and momentum variable (‘cause we have energy conservation, and momentum conservation; further ‘cause collisions should relax the energies of the two guys to same temperature, and momenta of two guys to same entropic velocity). But then we’d miss knowing exactly how the momenta and energy relax if we presume this, so. So it’s just easier to do it like this I think. Note ℘ shows up in s here because we do not have impurities which would try to ‘relax’ the velocity distribution of any local piece of our fluid back to zero, and so it should constitute a real equilibrium thermodynamic variable. And I’ll let ψ be some constant external field, whereas **F** will be some changing, non-conservative force. I’ll suppose that whatever non-conservative force is out there is sufficiently smooth so that it is constant over υ and τ, and so can be pulled outside the averages. I guess **F** would have to be the force coming from any other particle. I’ll ignore angular momentum contributions as a liquid/plasma cannot exert shear forces I don’t believe. And I’ll ignore internal structural energy changes. So then I’ll say the balances for particle species one and two are, respectively,



and,



where the Δ terms, Δ and Δ are due to interactions/collisions, say, between the species within the differential volume υ (and they are position/time dependent) that we’re statistically averaging over (see Laws of Thermodynamics/Continuum). I’m explicitly invoking energy and momentum conservation when writing them with opposite signs between the two species. Note π1 isn’t equal and opposite to π2 though – the π’s basically tell us the rate of momentum diffusion across a surface, and one species doesn’t have to diffuse the same as another, especially if one is more dense than the other in thar region. Or if you look at it from the perspective of a force on an imaginary partition between the species, the forces don’t have to be the same: the difference would be the rate of change of momentum of the partition. And then the entropy balance would look like this:



where s(**r**,t) is s1(**r**,t) + s2(**r**,t). Well, let’s get to it. Plugging the balances into the entropy balance, using (m = 1,2 standing for each of our fluids) Tmsm = (εm + pm - **℘**m·**v**m – μmnm), and dsm = (1/Tm)dεm – (**v**m/Tm)·d**℘**m - (μm/Tm)dnm – (mm/Tm)dψ (where mm = qmnm and qm is the mass or charge of each particle, depending on the field we’re dealing with). I guess I’ll have to split this up into LHS and RHS. So, LHS is given by….using Einstein summation notation by the way:



(blue terms cancel out with each other, and red terms will cancel with the red terms below) and the RHS,



Now solving for sint…and simplifying,



Each of the colored terms sums to zero. And we can simplify the jq and π terms, and also going to change indices i→j and j→i (can do because just dummy indices) in the first two of those ℘ terms…



Then continuing, can see those ℘ terms cancel as well. This leaves us with:



Or back in vector language,



So let’s apply these considerations to a few models of interacting electrons/ions. So the first guy is just two copies of the single fluid terms we found in last file, and the last two are additional terms stemming from collisions between the two sets of particles which would ultimately encourage them to equilibrate to the same temperature (‘cause energy conservation) and center of mass velocity (‘cause momentum conservation). I think these probably happens on slower time scales than the other guy? Often it seems the last two terms are neglected.

**1. Fixed substrate (no dissipation)**

Now I’d like to develop a set of equations that describe the aggregate motion of free charges. I’ll start with the assumption of no dissipation. Putting this in the context of electrons on a fixed substrate of equal and opposite charge, the equations for the electrons would look like this (assuming no external field ψ):



As we did in the Navier-Stokes file, we can reduce this to (setting **F** = -e**E**):



where **E** is the electric field, and *e* is the absolute value of the electron charge. Turns out we won’t need the energy equation. If we assume that we can model the charges as behaving as an ideal gas (obviously their interaction w/ each other would militate against that assumption), then we will know s(ε,n) → p(ε,n) → p(n,s). And so this will be enough equations to ascertain the motion of the charges. Note that the pressure term must be there, even if the gas is dilute, and one ‘supposes’ that there is negligible pressure exerted. And this is because in this context it would constitute diffusive momentum flux. Now I’ll assume that we have free charges on a fixed substrate of positive charges. In that case, the electric field can be related to the charge density as in EM.



and the pressure, assuming ideal gas (note it may not be totally unrealistic to presume ideal gas since dense balls of charge have much more KE than PE due to Fermi exclusion principle, and plus, the PE might be minimized due to screening), would be:



Taking the divergence of the middle equation we’d have:



And so this set of equations is, as we can see, complete. But for simplicity, let’s expand the quantities to first order.



We’ll note that ne,0 = n0 = average density of electrons/ions. Filling these into the equations, we get:



Taking the FT of both sides, in space and time, we’ll get:



Now I’ll substitute the outer equations into the middle one,



Now p0/n0 = kBT. So can write this as:



So we get the dispersion relation,



The first term is just the plasma oscillation frequency (zero wavelength), and the second is just the familiar sound wave term. But yeah, so:



And evidently, we can have plane wave excitations through this medium of the form n1(x,t) = exp(ikx-ωkt). Can see the long-range nature of the e-e interaction supports non-zero frequency oscillations in the limit of long wavelength (small k). If we used a screened e-e interaction, I presume these oscillations would look more acoustic in form, i.e., ωk = ck. Well, not exactly, because the ions wouldn’t be good at screening the electrons’ motion. Note these oscillations are the same we find the Electrodynamics/Metal Physical Model file, the Stat Mech/RTA Susceptibility file, and in the Cond Matt folder/Metals/e-e Interactions/Nearly Free/Excitations/Collective Excitations file. There we found (will find) that we get plasma wave excitations with dispersion of the same general ωk = √(ωp + const.×k2) form. Only difference should be that here we used the equipartition theorem to evaluate thermal expectations instead of there where we’ll use the low T Fermi distribution. But a plamsa is typically at super high T, so our use of classical statistics here is probably okay.

Another thing of note is that this tells us why electrons don’t propagate sound, while the crystal lattice does. For a sound wave to propagate through a medium, we’d need to be able to write down n(x,t) = eikx-iωt in each region and match up boundary conditions, etc. In particular, we need the frequencies to match in each region. But ωk > √(ne2/mε0)~ 1015Hz, which is way larger than any sound frequency. So we can’t possibly match frequencies. But the lattice can. Since electrons screen the ion-ion interaction, it becomes effectively short ranged, and so that spectrum is linear like ωk = kv. And so for any sound wave frequency, there will also be a lattice oscillation frequency to match. Also, I don’t think we should worry about the outside air not being able, physically, to induce oscillations in field of electrons. If it can induce oscillations in a field of lattice ions, then I imagine it can in the electrons too. Or even if not, once the lattice ions are perturbed, then *they* would perturb the electrons.

**2. Free substrate (no dissipation)**

Now let’s suppose that the fixed positive lattice is no longer fixed. But it seems that we'll be assuming no collisions occur between the electrons and positive (ionic) charges, and so still no dissipation. So we’ll have the same equations, except supplemented now by the continuity equation for the positive ions.



So we have 6 + 2 equations, and 4 + 4 unknowns. So we’re good. Now let’s take divergence of the momentum equations, like before:



We’ll expand to first order again.



Filling these into the equations, we get (for electrons)



and similarly for the ions we get:



Now I’ll reverse order of operations from last time. Instead of taking the FT of both sides, I’ll just fill the top and bottom equation into the middle one, yielding (for electrons):



(note the replacement of s1 with (s0/n0)n1 doesn’t strictly follow perhaps; but it would if we can assume they’re both sinusoidal functions. Doing same with ion equation,



And here we’re kind of stuck. I’ll take the Fourier transform again,



Simplifying both,



So getting the electron density by itself, we have:



The solutions for ω gives us the dispersion relation. It’s certainly interesting looking. Note that we could obtain a decent approximation by neglecting the meω2 term. Except for the e2n0/ε­0 term, we’d just have two copies of the prior equation. So we’d have plasma oscillations of electron ‘gas’, as before, and now additionally similar oscillations of the ionic substrate. But with that term, these two modes do mix, to give us something slightly different; I’d imagine it’s some kind of oscillation about their mutual center of mass.

**3. Free substrate (allowing dissipation)**

Now let’s be more realistic and allow collisions to occur. I’m still going to assume that both sets of particles are at the same temperature, and that the **π** tensor is given by p**1**, etc., so that the velocity gradient term in σ disappears. Our equations will be,



So the last equation implies, from Onsager’s relation,



With this constitutive relation, we now have enough information to solve for the motion of each species. But we are particularly interested in the motion of the center of mass, **v**, of the stream of particles and the net current, **j**, however. So if we define ρ = mini + mene, **v** = **v**cm, n = ni + ne, and **j** = eni**v**i – ene**v**e, we can recast the first two equations in terms of these new variables.



and then we’ll do some work on the two momentum equations. First I’ll add the momentum equations together, neglecting products in **v**, and setting the total pressure p = pi + pe.



Now we multiply the momentum equations by q/m and add them together, again neglecting **v** products, and time dependence of n’s.



Now we’ll put the velocities in terms of the center of mass velocity and the current.



Then we’ll assume that mi >> me, ni ~ ne = n/2, giving us:



Now we’ll put these in our equation, also going to assume pi ~ pe ~ p/2.



Next we’ll fill in the Onsager relation





and these are our equations,



Might compare this equation to the one we get in the Stat Mech folder/Classical NESM RTA (MFT) file, although, in that file we presumed a stationary positive jelly substrate, instead of a mobile one. And we also presumed dissipation through disorder, not through collisions between the electrons and positive substrate.