**Perturbative Expansion of GF**

Going to ignore until/if necessary the implicit exponential convergence factors attached to the θ’s, where present.

**Example. HO in constant external field**

Let’s take simple case of this guy – a harmonic oscillator in a constant external field:



We found in the single particle QM file that:



where,



and that:



It just changes the zero-point energy, as we’d expect from classical mechanics. Let’s get some GF’s. I’ll skip some steps, using results from the non-interacting GF file.

**Exact Greater GF**

Let’s do the greater GF.



where



**Exact Lesser GF**

Let’s do the lesser GF.



**Exact Causal GF**

Now let’s do the causal Green’s function.



**Exact Anti-causal GF**

The anti-causal Green’s function can be obtained in similar fashion.



**Exact Complex time GF**

This is:



Basically just take GC(t → -iτ). So that checks out.

**Exact Retarded GF**

The retarded Green’s function is relatively simple. Working it out we get,



interesting that thermal averaging has no effect on the retarded GF.

**Exact Advanced GF**

For this we have:

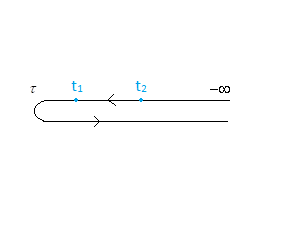


**Perturbative expansion of causal GF**

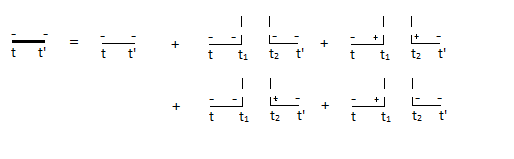
Now let’s consider the perturbative expansion of the causal GF. This would be:



(remember HO has no chemical potential) where the contour is:



I’ll presume the denominator just cancels vacuum bubbles as usual – though it would seem to me that the denominator itself should just be 1, but whatever). In diagrams, all we’d have is:



Mathematically, this is:



There are no ½ factors on the - - and ++ vertex guys because the external legs make them not interchangeable. Putting everything in terms of >, <, we get:



This must be independent of τ. So let’s see how that is true. The terms grouped together ought to do the job.



So then,



and further,



Now let’s fill in these functions. We’ll recall from the non-interacting file, that:



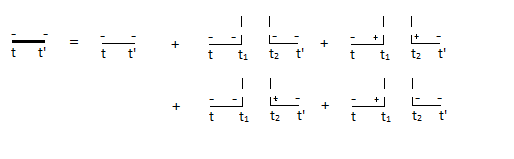
where we’ve included the exponential convergence factor that was left implicit. So,



(I just didn’t bother with the lower limit because the exponential factor kills it) and so,



So that’s correct…. It’s simpler to do everything in Fourier space. Going back to:



we’d have:



and if we look at G0>,<, we can see that they’re equal to zero when ω ≠ ω0. So then we have:



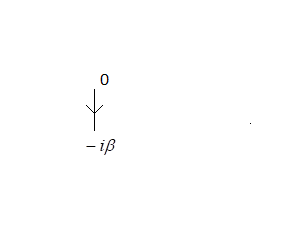
which is also correct.

**Perturbative expansion of complex causal GF**

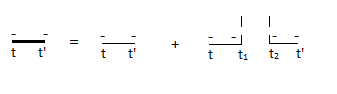
Now let’s consider the perturbative expansion of the causal GF. This would be:



and the contour is:



Vacuum bubbles cancel again. In diagrams, all we’d have is:



Mathematically, this is:



Filling our unperturbed G’s in,



Well I had worked this out to the end, but Mathtype corrupted the file so whatever. Suffice to say, despite what it looks like, it *will* match the exact result.



Of course we must explicitly use the fact that n = 1/(eβω0-1).