**Dielectric Susceptibility**

Want to consider the RTA model approximation for working out some time-dependent things we proferred in the EM folder, like the conductivity and dielectric susceptibility. So,

**Metals**

Let’s examine the susceptibility from the perspective the MFT solution to the RTA equation, which we developed in a previous file. So we found these two self-consistent equations:



where D was given by:



The susceptibility, χirr, is defined as the charge density response to an electric field perturbation.



But in the faux-Gaussian units we’ll be working in, where ε0 → 1/4π, we’d have:



But…then it’s customary to redefine the susceptibility by absorbing the 1/4π into it. So we’ll say,



We can work this out from our equation. Let j = 0, and B = 0 too. We’ll assume we have an isotropic medium, so **D** = D**1**, and we’ll call ρind = en. Then we have:



Now we can’t exactly lift the gradient from both sides because n in the numerator is presumably position dependent, as ρ is clearly supposed to have a non-zero gradient. But if we restrict our aspirations to just getting an equation for ρind correct to first order in φ, then we can treat that n as a global constant. And if so, then we can say,



So we have:



Now we’ll fill in D, the 3d, T = 0 version,



Now use the 3D result (see Condensed Matter/Free Day/Electrons/Properties),



And we can say,



So we arrive at the so-called Thomas-Fermi susceptibility.



We’ll get this result again from a different approach in the Condensed Matter/Free Day/Electrons folder. We can define a generalized susceptibility. Say we have a potential perturbation φ(r,t) = Re[φ(q,ω)eiq·r-iωt], and an induced charge density respone ρind(r,t) = Re[ρind(q,ω)eiq·r-iωt], then electric susceptibility is defined via the proportionality:



We can work this out, but we’ll need the time-dependent RTA equation – still no B field. And we’ll take the low scattering limit, i.e., τsc → ∞. In this case, the non-zero/dominant terms are:



Well we need to relate jind and ρind. We have the continuity equation,



So let’s take the divergence of both sides of our RTA equation,



Then we can say,



Now fill in our expressions for ρind and φ,



So we have:



Filling in D = 2τscεF/3m again, we’ll get:



where we use εF = (1/2)mvF2. Now also use, from Condensed Matter/Free Day/Electrons/Excitations/Properties file, and elsewhere, that kF = (3π2n)1/3 → n = kF3/3π2,



So we have:



The dielectric function follows (see e.g., Condensed Matter/Free Day/Electrons folder).



Okay, well, what if we leave scattering in there? Then we have:



Well we need to relate jind and ρind. We have the continuity equation,



So let’s take the divergence of both sides of our RTA equation,



Then we can say,



Now fill in our expressions for ρind and φ,



So we have:



Filling in D = 2τscεF/3m again, we’ll get:



and so we end up with,



Before we move on, l should mention that σeff(q,ω), which incorporates the effects on the current from **E** directly through its force on charges, and **E** indirectly through charge density fluctuations, and which is given by:



is proportional to χirr(q,ω), which is given by:



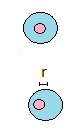
In fact,



Can see the EM folder/Metal Model (TD) for discussion on this point. Now let’s look at insulators.

**Insulators**

Now we’ll do insulators. For reminders, in our simple classical model, we have an electron shell centered over a positive nucleus:



which exerts a position-dependent linear restoring force on it (as derived from Gauss’s law)



In addition, we’ll presume there is some ambient electric field **E**(t) exerting a force on our electron shell too. Let’s go back to our Boltzman equation. We’ll presume τ is just a constant.



We want to work out <**r**(t)>, because we want the polarization vector <**P**(t)> = -e<**r**(t)> (e is positive). So let’s multiply both sides of the Boltzman equation by **r** and integrate.



So that’s part of the way. But now we need <**k**(t)>. So we’ll multiply by **k** and integrate…



At this point, we’re kind of stuck. But we can close the two equations by presuming the distribution function f(r,k,t) is highly peaked about the average position. Something like f(r,k,t) ~ δ(r-<r(t)>). Then we could say:



Filling in we our force formula (e is positive), we have:



This is just two equations and two unknowns so we’re good. Somehow. Since we’re just looking for the steady state response, we could take the Fourier transform of each side. Then we have algebraic equations. So then could solve for <r(ω)> and take inverse transform. But I think instead we’ll make another approximation: that the rate of change of momentum coming from scattering is a lot smaller than the rate of change due to the forces acting acting on the particles. If so, then we can say:



So then we’d take derivative of top equation and plug the bottom equation into it.



Now we’ll reprise our calculation in the EM folder. We solve this equation by taking the FT of both sides (γ = 1/τ):



Well, now, the polarization (density) is:



where n is the number of dipoles/molecules per unit volume. And so the Fourier transform is:



So it follows that the polarization is:



and there upon, the susceptibility is:



So there we go.