**Boltzman Equation w/ Two-Particle Potential**

Now going to look at incorporating quantum mechanics.

**Semiclassical Boltzman Equation for two particle potential**

We can incorporate a little quantum mechanics by accounting for the Pauli exclusion principle. If we’re dealing with fermions, we just have to multiply our terms by the probability that the states the fermions would scatter into are empty, and can therefore accommodate that scattering process. So let’s go back to the discrete Boltzman equation:



And update to include the aforementioned factors. The probability that none of the particles is in the state chunk centered at r,k,σ would be 1-Nfσ(r,k), as fσ(r,k) is the probability that a *given* particle is at r,k,σ. Should probably be using N-1 rather than N, since we really want the probability that the state is occupied by particles other than the one presently scattering. But difference shouldn’t matter for systems of interest.



And now let’s put in terms of f and W densities: fσ(r,k,t) → fσσ(r,k,t)Δ3rΔ3k and W(kσk1σ1→k´σ´k´1σ´1) → W(kσk1σ1→k´σ´k´1σ´1)Δ3k´Δ3k´1, as we did above.



Canceling stuff out,



Now I think we can use the fact that in our semi-classical approximation the phase space volume Δ3rΔ3k (or Δ3rΔ3k´, etc.) are fixed to (2πℏ)3, which in our units is just (2π)3. So then going to the continuum, we have:



And presuming TRS, we got:



If we presume the scattering rate is independent of spin, then we get:



So in that case, we have:



**Putting in terms of kσ occupation numbers**

So often the Boltzman equation is not written for fσ(r,k) as a continuous probability density, but for nkσ(r) as a discrete occupation number [per Δ3rΔ3p = (2πℏ)3] function of the quantum states rkσ.



It has the properties,



using Δ3k = (2π)3/V. And averages would be done via:



So let’s make that conversion for later use. We’ll go back to the quantum continuum result.



And convert the k integrals to sums,



Now remember in the discrete case W(kσk1σ1 → k´σ´k´1σ´1)Δ3k´Δ3k´1 is itself the scattering rate. So we’ll make that change back: W(kσk1σ1 → k´σ´k´1σ´1)Δ3k´Δ3k´1 → W(kσk1σ1 → k´σ’k´1σ´1).



Now use Δ3k1 = (2π)3/V, and change variables to nkσ(r) = (2πℏ)3Nf. Then,



And canceling out stuff,



To first order in perturbation theory, W(kσk1σ1 → k´σ´k´1σ´1) would be, in analogy with single particle stuff:



Here’s some more on the relationship between f and n,



where Δ3rΔ3p = (2πℏ)3. Last line is kind of fishy.