**Magnet**

**Prologue**

Now we want to investigate how we may determine the functional form of these potentials introduced previously. We will consider for simplicity that our substance is homogeneous and obeys the scaling relations introduced in the previous lecture. Then our potentials satisfy,



It will suffice, then, to determine the heat capacity CV(T,V) and equation of state p(T,V), to construct the potentials [these basically give us S and p for the free energy F]. For instance, forming the differential of U(T,V) we have:



What are these derivatives? Well,



and,



So we have:



A similar procedure for the entropy would yield:



What are these derivatives? Well,



and,



So we have:



With U(T,V) and S(T,V), we can get F(T,V) = U(T,V) – TS(T,V) for instance, which is one of the thermodynamic potentials. So from this we can construct everything we want. And of course we can invert these equations to construct other potentials as desired.

**Magnetic Materials**

Now let’s consider a magnetic material. A magnetic material’s energy levels depend on the ambient external (free) magnetic field, Bf = μ0H (assuming solenoidal geometry and homogeneous substance we can equate this two). So En(Bf). And so S(E,N,Bf). And so Bf takes the place of V in our previous formulas (well we could still have V as well, but we’re neglecting it). So we’d have the following definitions, analogous to our previous materials…gonna call Bf = h.



We should expect that E(λS,λN,h) = λE → E = E(S,1,h) → dE = TdS – Mdh. Measurements taken on paramagnets/ferromagnets established the following results, valid for small fields and/or high temperatures (heat capacity is that due to just the magnetic moment d.o.f. – can check out mean field calculations for paramagnet in Stat Mech folder for comparison).



where α and β are just some phenomenological parameters. So what would the potentials be? Let’s go through the process again. We have:



But we need to get a differential E in terms of T and h, rather. So we look at the Free Energy,



Its differential is:



And so we have:



And derivative w/r to h is, at constant T,



So then we have:



Filling in our measurements,



We don’t necessarily know what Ch is, but note that much of it is already fixed by the equation of state, since we’ll note that cross partials must be equal. So this requires:



So that tells us α = β. Resuming the E calculation:



Integrating ∂E/∂T = α(h/T)2 w/r to T and then differentiating that w/r to h and setting to -2α(h/T),



leads us to conclude:



Might note that we could write this as: E = E0 – Mh. Now for the entropy. Now that we have E, we could just get S via S = (E – F)/T. But we’ll do it this way. So,



What are these derivatives? Well,



and from , we can see:



So we have:



Filling these guys in, we find:



which implies:



Interestingly, S → const. as T → ∞. This would indicate that there are a finite number of states for the system to occupy, especially I guess that there is an upper bound to the energy states it can occupy (and we know there is – the spins max out energy when pointing opposite the field). That’s what kind of makes it max out its randomness as T increases. Another interesting fact is that we can write S purely in terms of M. We have: S(T,h) = S0 - (α/2)(M/α)2. Interesting implication of this is that if we were to insulate the magnet against heat transfer, and increase the field, its magnetization wouldn’t increase (because S wouldn’t increase). So can think of it this way: increasing the field does work on the magnet, which would increase the alignment of the dipoles with the field and thereby increase M, except that it also increases temperature, which increases randomness and thereby decreases M. Apparently these two cancel out. This is similar to what happens with ideal gas in a box. Compressing gas reduces volume, which would tend to reduce entropy, but also increases temperature, which increases entropy – and it all cancels. Maybe calculate the Free Energy,



Can see M = -∂F/∂h, as it should. Wanna check out the 1st law for processes at constant temperature. Particularly, want to see what dW is:



And this accords with dW = -Mdh, as it should. So in a constant T processes, the energy is decreasing, because both work and heat transfer are negative. Usually it’s said that when increase magnetic field through a paramagnetic substance (and presumably at constant T whereby the magnetic susceptibility is constant), positive work is being done on it, and usually justified via formula dW = hdM. And since we’re doing + work, that would increase T, and then need to extract heat to keep T constant. I’m thinking this is wrong though? I think dW = -Mdh, as used above, because it fits better with other things. And so then we would just have to admit we’re doing negative work on the substance, but not so negative as how much the energy is decreasing, and so we still need to extract heat to keep T constant.

**Example**

Starting from the beginning, say we have a magnet, and we measure the following,



Fill this into:



Equating cross partials, we have:



So,



Filling this back in,



Then we can get the energy. Integrating w/r to T,



Differentiating w/r to h,



So up to a constant, we can say:



**Example**

Starting from the beginning, say we have a magnet, and we measure the following,



where α is known, but β and n are not. What are they? And what is E(T,H)? What is S(T,H)? Fill this into:



Equating cross partials, we have:



So,



So we have:



Filling these back into the energy,



Then we can get the energy. Integrating w/r to T,



Differentiating w/r to h,



So up to a constant, we can say:



What if we want the entropy? Well,



So, integrating w/r to T,



and differentiating w/r to h,



So,



**Example**

Starting from the beginning, say we have a magnet, and we measure the magnetization at constant temperature and find:



a is not known. It is presumed, however that the entropy should be purely a function of the magnetization. What is a? And what is the heat capacity?



Filling what we know into this, we have:



So, integrating w/r to h, we have:



Comparing with:



we see f(T) must be a constant, and a + 1 must be (3/2)a. So,



Therefore,



And the entropy must be:



and the heat capacity would be:

