**Boltzman Equation w/ Two-Particle Potential**

Now let’s go back to:



and consider its various solutions.

**Solution to Equilibrium Equation**

So earlier in the RTA file, we looked at the scattering-less Boltzman equation. And we looked for possibile equilibrium distribution functions. Such a distribution function would have to be time-independent and also satisfy the Boltzman equation of course. We verified that a set of solutions was of the following form:



For instance, apropos the first guy:



where F = -∇φ. What about the RHS? Local equilibrium solutions are ones which annihilate the RHS of the Boltzman equation (or, trivially, the RHS of the RTA equation). In the distinguishable particles case, this would require:



This tells us that lnfleq(r,k) + lnfleq(r,k1) must be a conserved quantity before and after the collision. There are three things: momentum, energy, and particle concentration.

And so we’d have to have something like,



where we absorb a and **b** and c into new constants β, **u**, and c. β is of course 1/kT, not that we’d know it yet. And **u** is the overall average velocity of the particles, like if we have a gas in a box in a car. So altogether we’d have:



The constant c would be chosen to put the distribution in the standard form,



The chemical potential would be chosen to normalize it, so as to make:



These fleq’s would annihilate the RHS and so are local equilibrium functions. But they are not global equillibrium functions unless they annihilate the LHS too. So that would restrict global equilibrium functions to the form,



where β, u, and μ have no spatial/temporal dependence. So there. Couple more comments. So if we were to include the single particle scattering term here too, then the local equilibrium solution would be of the form,



i.e., we must take out the convective velocity term. This is evinced from the fact that momentum isn’t a separately conserved variable. Can see the Stat Mech/Boltzman 1PI file for more details on that. To reach back to the RTA file, we can say that this is where we find the justification for the form of the fleq guy in the ad hoc (fleq – f)/τ term we put on the RHS of the RTA equation.

**Perturbative Solution to Non-Equilibrium Equation**

Now we’d like to consider non-equilibrium solutions to the equation, where we allow for presence of non-conservative force fields, and temperature, chemical potential gradients, and velocity gradients [remember the chemical potential depends on r and t implicitly, through potential, density and temperature as it must be such as to satisfy the equation n(r,t) = N∫d3k f(r,k,t)]. This will be hard to do so we’ll look for a perturbative solution, first order in temperature and velocity gradients. But because the RHS is a non-linear integral operator, solving even the first order equation will be harder than it was in the 1PI case. Anyway, we’ll postulate f = fleq + δf = fleq(1+χ):



where this fleq(**r**,**k**,t) is a *local* equilibrium distribution function. Note such a form for fleq does annihilate the collision integal, but does *not* satisfy (LHS)fleq = 0, just as was also the situation for the RTA Boltzman equation. And so unlike the set of equilibrium functions in the previous section, it is not an actual steady state distribution. This is especially so since it does not describe any entropic currents. Plugging this into the BE and keeping only the largest terms on each side (basically ignoring all gradients on χ as already second order), we have:



Fleq annihilates the RHS of the equation, as we found above, and just as it did in the RTA. Now recall from above that:



This enables us to simplify a bit,



Dividing both sides by feq(**r**,**k**,t), we get:



Now define the operator , as:



Recalling,



the operator simplifies somewhat to:



Now also define the inhomogeneity Y as:



and we can write our equation as,



We can work out Y a little (using units where kB = 1). Using,



we have:



And well, proceeding,



Fc is conservative force, Fnc is non-conservative, and F is total force. Most of the time I think we can say T, **u**, μ, and φ are all time-independent, and Fnc too. In that case,



note (k-mu) dots into ∂u/∂r, and then k/m dots into those.

**Eigenfunction Expansion**

One way to solve our χ equation is eigenfunction expansion. We’d work out the eigenfunctions of L, φn(**r**,**k**) [this φ obviously not the single particle potential….], and then expand χ and Y in terms of them. Note L only operates in k-space. Will presume that feq is time-independent for this. The eigenfunctions would work like this:



I think r would just be considered a parameter kind of. And the orthonormality condition would be:



Note the presence of fleq(r,k) in the inner product, which is a weight factor necessary to make L Hermitian. So to proceed with the solution, we dot both sides of our equation with φ:



and now expand χ in a series:



so we get:



So this is our inhomogeneous eigenvalue equation. If we further presume that Y is independent of time, because I don’t want to bother, the solution would be:



Well I guess we can solve it so:



And our general solution would be:



where,



We run into an obvious problem if λm(**r**) is zero. And there are five eigenfunctions with zero eigenvalues. These are the (unorthonormalized) collisional invariants,



The latter φ(r) is the potential energy of course, not an eigenfunction. Anyway, as can see, this eigenfunction approach will only work if Y is orthogonal to these guys.

**Variational Approach**

Another way to work out the time-independent solution is with a variational approach. So here we’re looking for the steady state response. So let’s presume fleq, Y, and χ are all independent of time. Then we have:



Now consider <Φ|LΨ>. Want to write it in nice form:



where we define ΔΨ. Which we’ll write for short(er) as:



Using the fact that W(a→b) = W(b→a), and that fleq(r,k)fleq(r,k1) = fleq(r,k´)fleq(r,k´1), one can show,



This tells us two things:



The first establishes the adjointness of the operator w/r to the fle

q weight factor. The latter fact especially allows us to construct a variational principle solution to the Boltzman equation. Consider a solution, Φ, to our equation:



And another function Φ + δΦ. And consider the functional:



Then,



So,



So an equivalent formulation of our equation is:



**Semi-classical Solution to 2PI Boltzman equation**

So the semi-classical equation is:



Let’s look at the local equilibrium solution to this equation. In this case we need,



And so we need this ln() term to be a collisional invariant. Like above, energy, momentum, concentration, and any local function of r, are all collisional invariants, so we must have:



which implies,



We can choose c to write this in more standard form:



The chemical potential would be chosen to normalize it, so as to make:



More generally, if we also handled bosons, we should find:



These fleq,σ’s would annihilate the RHS and so are local equilibrium functions. But they are not global equillibrium functions unless they annihilate the LHS too. So that would restrict global equilibrium functions to the form,



where β, u, and μ have no spatial/temporal dependence. So there. Couple more comments. So if we were to include the single particle scattering term here too, then the local equilibrium solution would be of the form,



i.e., we must take out the convective velocity term. This is evinced from the fact that momentum isn’t a separately conserved variable. Can see the Stat Mech/Boltzman 1PI file for more details on that. To reach back to the RTA file, we can say that this is where we find the justification for the form of the fleq,σ guy in the ad hoc (fleq,σ – f)/τ term we put on the RHS of the RTA equation. Anyway, as far as the non-equilibrium solution to the semiclassical 2PI Boltzman equation, I imagine we can do something similar to what we did for the classical version, but I’m not going to bother. Nope.