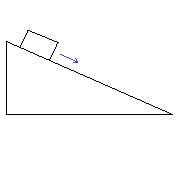
**Various Mechanical Examples**

Still going with some more examples….

**7. Friction**

Suppose a block slides down an inclined plane. What will be its speed at the bottom? What would be its temperature. Let’s assume it doesn’t change volume any.



The momentum balance in the x-direction would read:



where I’ve integrated just to show that the typical work-mechanical energy equation follows from the momentum balance. So this tells us the speed of the block at the bottom. Now let’s consider the energy balance. As discussed somewhere maybe, the friction force doesn’t do work per se´ (even the kinetic friction force), because the point of contact between the surfaces usually do not move. Instead, the friction force ‘catches’ points on the block, thus slowing it down – it’s like how if push yourself off a wall, the wall doesn’t do any work on you since the point of contact between wall and you is your hands and your hands don’t move. So there are no non-conservative forces present, in a sense. Nonetheless there could be work done via air pressure and volume expansion of the block, but ignoring all of that for the sake of argument .... we will have:



Combining this equation with the momentum balance, we can infer that:



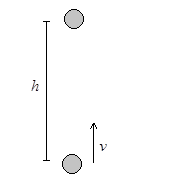
Now let’s consider the entropy balance of the block. Clearly since ΔU is not zero, neither is the entropy change. I’m going to imagine that the plane is rigid and the block not rigid, according to the usual model.



and I believe dSint. isn’t zero because of the oscillations induced into the block due to it ‘catching’ on the imperfections of the plane. We should expect dSint to be present in an irreversible process like this. And then using the entropy balance we can infer that equilibrium will be re-established eventually. Then we can use the entropy equation to express the energy in terms of the temperature, U = NkfT/2, and use the energy balance to determine the new temperature. A similar analysis would follow for the case of a ball dropped onto the ground. Its internal energy increases because of transfer of mechanical energy to internal energy. And this is how entropy increases too. In neither case is ‘heat’ delivered to the ball from the ground per se´ (at least not at first – on contrary, *after* ball heats up, then it will deliver heat to the ground).

**8. Air resistance friction**

Suppose we throw a ball up into the air with initial speed v = 10m/s. How high will it go before coming back down? What can we know about its change in temperature?



The momentum balance would read:



And if we want to determine the height of the ball, we can use the momentum balance for that. Standard integration yields:



For the energy balance, recall that the force Fair doesn’t actually do any work on the ball. This might seem implausible. But I think we might be able to justify this statement by saying that the work that air would do on the ball would be pressure work, like in the buoyant force. But here this force resulting from turbulence does not do work. So then we have:



We can then write:



and filling in the momentum balance result:



Note how in the limit that γ → 0, we will have ΔU = 0. The entropy balance would read, assuming no heat transfer in this simple case:



Again, using the entropy equation we may determine a formula for T in terms of E and therefore infer the new temperature.

**9. Minimization of mechanical energy**

Why does block slide to bottom of hill and stay there? Under what conditions is (mechanical) energy minimized?

Well system is closed per se´. So of course entropy will be maximized. Entropy is given by S(U,V,N) = S(E-Emech.,V,N). V and N are fixed. But if we minimize Emech., then S will be maximized since S ~ U, as ∂S/∂U = 1/T which is positive. Note that this implicitly solves the question I used to have over whether you should minimize Emech. or F to find the shape of a bubble. Answer is you should minimize F of course – but doing so requires minimization of Emech., which requires minimization of PEg.

**10. Bernoulli’s equation**

Derive Bernoulli’s equation. We’ll consider a control volume fixed in space. Then the balances are:



and, assuming no heat flow, no change in energy i.e. steady state:



Using the particle balance to cancel out like terms, we have:



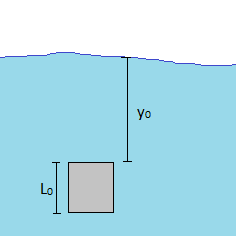
so there we go.

**11. Starbucks**

Consider shaking a GTL. If you push it upwards and then stop it suddenly, you will give it kinetic energy obviously, but you’ll do no work on the liquid as you slow the cup down, since the liquid will keep going until it hits the lid (unless severe friction between liquid and sides of cup). When it hits the lid, its kinetic energy will be dispersed into the bulk, and its temperature will increase. Can calculate this change. Can do same if you drop the GTL onto the floor.

**12. Air bubble**

Let’s consider an air-bubble which we’ll model as an air-filled box with rigid sides but moveable top and bottom. And let’s suppose it constantly stays in thermal equilibrium with the water surrounding it, at temperature T. If the bubble has height L0 initially, and starts at depth y0, how long until it reaches the top. And what will be its height?



Then the global momentum balance yields:



The global energy balance would look like this:



and then we have the entropy balance,



I would assume that Sint. = 0 and so the I could solve for dQ/dt in terms of dS/dt. Plugging this into the energy balance we have:



So then these two equations could be solved for y, in principle anyway.