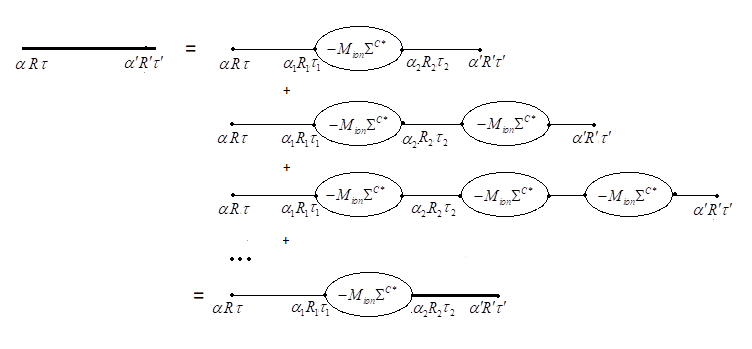
**Self-Energy**

We can construct a self-energy, at least in matrix form if we have index/spin-dependent interactions. The way to do this through the real time formalism, via which we ultimately get an equation for GR was sketched out in the QM file. And the procedure here would be the same – just replace state averages with thermal averages. So instead of retreading that territory, I’ll just move on to the complex time GF. Once we get it, then of course we can find GR through analytic continuation.

**Distinct Particles on Lattice**

Let’s start with the self-energy on the lattice (note I defined the self-energy bubble as -MΣ, rather than -Σ, to make the final result look nicer)



which is equivalent to the equation:



Let’s say that we have a homogeneous system. Then G, and Σ would depend only on the difference R1 – R2, and τ1 – τ2. This would allow a spatio-temporal Fourier transform on our recursive equation. And we’d have:



where 1/ and -1 is to be interpreted as a matrix inverse operation. Might be best to recognize the top line is just a matrix equation (in polarization space), and the MGΣG term is just a product of three matrices. If we can presume isotropy too, so that G and Σ are diagonal in their polarization indices, well, proportional to the unit tensor δαα´. So then we can fill in our previously worked out non-interacting G0 and come to:



By analytic continuation, we could then get GR(**k**,ω):



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole, ω = k, defined by:



Suppose we find the root of this equation ω2 = [we’ll impicitly note that Σ is a function of ω2 because ω only appears in the G’s as squared]. Then we’ll expand the denominator of the Green’s function,



which we can write as:



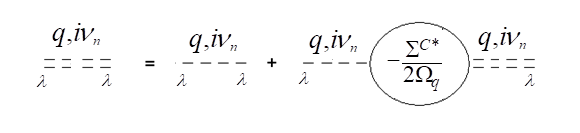
As usual, we can interpret the real part of Σ as the energy shift (or well, contributing to it), and the imaginary part as inverse lifetime of the quasi-particle state (contributing, again). And could get the spectral function, etc., like has been done before. Note the advanced GF would be given by:



Since we expect these two GF’s to be different, there must be a branch cut in the function ΣC\*(k,ω) along the ω = 0 line. In fact, I think GR = GA\*, so ΣR = ΣA\*. It is worthwhile to see the self-energy structure of the DλC\*(**k**,ω) Green’s function too. Recall that this was defined via:



The self-energy equation for D would look like this (now I’m representing the exact D with double dotted lines, and I’m propitiously guessing what the self-energy bubble is, in terms of Σ, as we’ll see below). I’m assuming the interaction, whatever it is, is homogeneous / isotropic / whatever so that different polarizations aren’t mixed (except within ‘Fermion loops’ where they would be summed over).



The recursion relation gives us:



which is:



which is, finally:



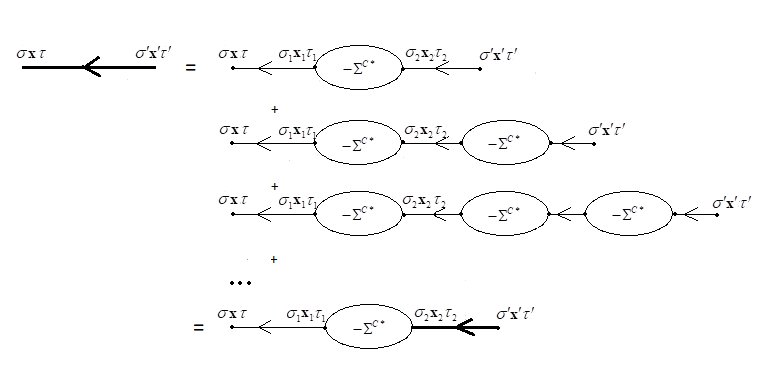
and if we plug this back into the equation relating G and D, we see we get our previous G result back,



So that’s good.

**Identical Particles**

Now let’s look at the identical partical complex time-ordered GF. We can write its expansion in terms of the self-energy.



In real space, we’d have quite generally:



and assuming V1 = 0, the GF’s will be a function only of the difference of their arguments. Then so will the self-energy. And so then the spatio-temporal Fourier transform will give us:



We can solve this (matrix in spin space) equation. Easiest to just recognize that this is a matrix equation in spin space. Temporarily getting rid of indices, and denoting G, Σ, G0 w/o indices, as matrices in spin space, our equation is:



where the -1’s are all matrix inversion operations. More cleanly, but less explicitly, we can write this as:



where as noted, 1/ and -1 are to be interpreted as matrix inversion operations. If we presume isotropy in spin space (no magnetic fields, say), then G, G0, and Σ are all proportional to the unit tensor δσσ´, and we can write this as, inserting our known result for G0C\*



where ξk = εk – μ. By analytic continuation, the retarded GF is given by:



We ought to think of these as functions of ω, with k constant. k simply determines the particle whose self-energy function we’re considering. Given this form, we can obtain the spectral function:



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole.



Suppose we find the root of this equation to be . Then we’ll expand the denominator of the Green’s function,



which we can write as:

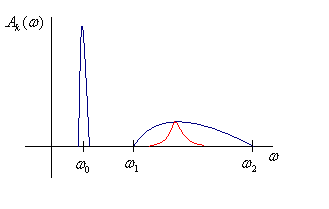


Z(k) is called the renormalization factor. Z(k) < 1 of course. And we would identify

as the energy correction, and as -Γ/2, half the decay rate. We can also examine the spectral function, which will give these identifications another light. The spectral function will come to:



and again, the identifications make sense. Note how ImΣ must be less than 0 for A to be positive. A typical spectral function, A(ω) will look like below, in blue



The two peaks indicates that there are two excitations of the system associated with momentum, k. The width of the peak is given by the ImΣ. So the delta function thing on the left would have that ImΣ = 0, while not so for the right hand curve. If we tried to approximate the latter with a Lorentzian, as we do above, we’d have the red curve, whose location gives the energy best associated with it, and whose width would give the scattering rate basically, and whose height would roughly give the renormalization factor, basically the amount that this would resemble a quasi-particle.

**Photons**

Now let’s look at the self-energy of the photon GF. Recall the free photon Hamiltonian, and let’s add some interaction to it.



In all cases of interest so far, it is Akλ, Akλ† which couples to other fields in the Hamilton. So the Green’s function was defined as [(H) means we’re looking at the fully time-developed operators]:



where,



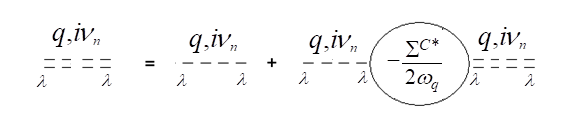
and this works out to (see non-interacting GF):



Fourier transform yields,



And in a homogeneous isotropic medium, the different λ d.o.f. shouldn’t be coupled together. So we can write a self-energy expansion for D itself. It would look like this, in spatial/temporal Fourier space:



The recursion relation gives us:



which is:



which is, finally:



and recalling what the sum over the polarization vectors equals,



(basically the two polarization vectors are an orthonormal set perpendicular to the photon’s velocity – because EM waves are transverse of course) the G green’s function could be written as:

