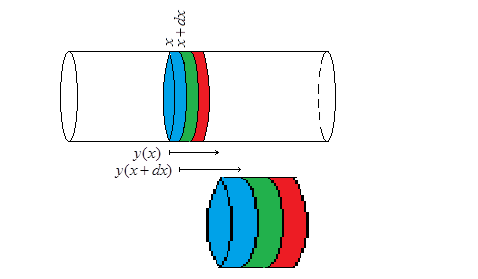
**Sound waves**

Going to work out the equation for sound wave propagation a few different ways and in a few different scenarios. This could be considered an extension of the last file stuff where we have interacting components in local equilibrium. Now the number of components strung together would be infinite.

If we want an expression for the time dependence of the system about this equilibrium configuration, we would have to assume that the system components are again in quasi-equilibrium (later called local equilibrium) and that the system’s fluctuations are reversible at this point, which would lead us to the condition. dS/dt = 0 → Σ∂S/∂**η**∙∂**η**/dt = 0. Then dS/dt ought to be zero so we set **F =** ∂S/∂**η** = 0. This is very similar to the non-equilibrium thermodynamics procedure, except that there we will set **F** proportional to d**η**/dt. Anyway, evaluating these equations should give us an equation for the reversible fluctuations, such as they are.

**Speed of sound via closed system analysis**

Let’s try to derive an equation for the velocity of a sound wave. Let us suppose that the air is separated by infinitesimally thin, and light membranes spaced at intervals dx.



As the wave proceeds through the cylinder, the membranes will be displaced according to the function y(x,t). Let’s say that each particle has mass m. Additionally, let us say that the fundamental variables of the slice between x and x + dx are N(x,t) = N, V(x,t), P(x,t) [P is momentum], E(x,t), and S(x,t). So what does thermodynamics says about the propagation of this wave? According to balance equations we have:



(apropos entropy balance remember that we have closed compartments so no entropy flux via particle flux) Dividing by Adx, we can write these as:



Note these variable symbols are a little misleading since they are the extensive variables divided by the slice volume Adx, not the actual volume of the compartment Ady. And so they’re not the customary extensive variable densities. Now let’s fill the balance equations into the entropy equation:



So we see that if we wish to consider simply reversible changes, then we must take this term to be zero. There are two options. In the first term we can set jq = 0 (isentropic process), or we can set T(x) to be constant (isothermal process). In the second term, our only option is to say π = p. Let’s go with the first option for now. Filling in what ℘, ε, and s are we have:



We have four unknowns, and four equations – so we’re good. We can use the entropy equation to relate p to u (or vice versa). But we have to use a little care because our u is U/Adx, which is sort of U/V0, not U/V. So we do the following manipulation….



So now have:



Now let’s just concentrate on the last two equations. We can use the 2nd to simplify the 3rd,



And now we can do cross-derivative stuff between this equation and the second to get p(x) by itself,



Which is quite a non-linear equation. We can linearize it by expanding p(x) about atmospheric pressure. Then we’d have:



Filling in p0 = nkT0, and u0 = n0kfT0/2, and simplifying, our equation reduces to:



So for small pressure variations we see that the oscillations will follow the usual wave equation with a velocity:



**Speed of sound via open system analysis**

Doing this from another point of view is instructive. We can imagine that our slices are fixed in position, and allow particles to flow into and out of our volume. So let’s do that. We’ll assume the usual notation for the densities. And make the same assumption about heat flow, and quasi-equilibrium nature of the interactions.



Expanding the entropy term and plugging in the balances, and using s = (ε + p - ℘v – μn) (see Thermodynamic Potentials):



Again, we want reversible dynamics, since we don’t yet know how to deal with otherwise. So we must set jq = 0 (isentropic processes), or ∇T = 0 (isothermal process), and must also set either π = p, or ∇v = 0. We’ll go with isentropic process again, as well as π = p. Can’t take ∇v = 0, for a sound wave at least, as this would not allow v to be a function of x, which it certainly would be. So let’s fill this stuff into balance equations,



Our solutions are of the form:



We’ll look for approximate solutions. Then density n(**r**,t) shouldn’t stray far from its average. So we may say n(**r**,t) = n0 + δn(**r**,t). Nor should the temperature. So we can say T(**r**,t) = T0 + δT(**r**,t). And pressure will constitute small deviations from its typical ambient value (p0 = 1atm presumably): p(**r**,t) = p0 + δp(**r**,t). Similarly, the velocity of the wave we might take to be ‘small’: v(**r**,t) = v0 + δv(**r**,t) = 0 + δv(**r**,t). The internal energy can similarly be expanded: u(**r**,t) = u0 + δu(**r**,t), where u0 = (f/2)n0kT0. So expanding the second equation to first order:



and third equation to first order:



Now let’s fill in fact that u = (f/2)p to get:



and then apply ∂/∂t to both sides of blue, and ∂/∂x to both sides of red, and adding appropriate linear combination of these we get:



which reduces to:



which is our equation from before. So that’s nice. Of course general solution is:



We can get pressure, density, etc., as well. From the blue equation, we have:



So pressure and velocity fluctuations are in phase. Consider what the first equation has to say,



So all these guys are in phase. How would we get displacements of a single particle from this? Well I guess this would be integral of δv(x,t)? That’s fishy.

**Example: Spherical sound wave**

Assuming no heat transfer, and spherical symmetry, what is equation for propagation of sound in air? Can’t really obtain this from the integral equation itself directly because if we integrate around a spherical shell we’re just going to get 0 in all cases I think. That’s why a set of spherical shell balance equations, done in the manner of the previous file, wouldn’t have really worked too well. OK now let’s work out the energy balance (again assuming no dissipation)



(Note that I believe this equation works out like a naïve spherical shell balance would have it do, and this is because it is a scalar equation, so adding all pieces of shell together don’t give 0). And note we filled in the expressions for p(r), u(r)…now for momentum balance,



Going out to first order in momentum balance,



Now doing the cross differentiating thing, multiplying top by k, and adding together we will get:



which simplifies to:



which is indeed our spherical wave equation.

**Example: spherical sound wave w/ gravitational attraction**

Alright last example. What would equation look like if we included a gravitational attraction between particles, still assuming spherical symmetry (and again, no dissipation)? Then the particle balance would yield:



The momentum balance would yield:



where g(r) is the local gravitational field. This field will be spherically symmetric under the stated assumptions. And moreover it can be determined from Gauss’s law:



Now the energy balance would look like,



and finally the entropy balance, assuming no heat transfer…



Now we can write the entropy density as:



where Φ is a known constant, at least from statistical mechanics. So in that case we have:



So altogether we have:



If we fill what p(r) and u(r) is then we get:



Can be simplified a little to:



keeping in mind that g(r,t) depends on n(r,t) too. Φ would be given by:



but it looks like we wouldn’t even need the entropy equation, as we only have 3 unknowns.