**Noninteracting Green’s Functions**

Now we’ll examine some non-interacting GF’s. Most are of the same form as those obtained before/only difference being a thermal averaged expectation replacing a single state expectation. Note I’m going to be presuming time-development starts at t = 0: So U0(t) = e-iH0·t. But if we used U0(t,t0) = e-iH0(t-t0) instead, this wouldn’t change G(t,t´) because it is, as you’ll see, a function only of the difference of the arguments. Also, for the most part, I won’t bother with the exponential convergence factors that we should have on the real space GF’s. Might check out QM/Time-Dependent Folder, and QM/Many Particles/Distinct Particles for more work on these, especially the Fourier transforms in the Time-Dependent Folder.

**HO Green’s Functions**

Consider the HO Hamiltonian,



In the QM folder, we saw that we could reduce it to:



where Ω is the oscillation frequency. And now recall,



Now let’s proceed, recalling a(t) = e-iΩta…some thermal expectations are:



where,



**Greater GF**

Let’s do the greater GF.



**Lesser GF**

Let’s do the lesser GF.



**Retarded GF**

The retarded Green’s function is relatively simple. Working it out we get,



Interesting that thermal averaging has no effect on the retarded GF. I’ll just just aver that if we take inverse FT of the FT (with convergence factors) then we get the effective G0R that we should be using in actuality. And this will end up to be:



Similar factors would show up on other GF’s (though not G>,<).

**Advanced GF**

For this we have:



**Causal GF**

Now let’s do the causal Green’s function.



**Anti-causal GF**

The anti-causal Green’s function can be obtained in similar fashion.



**Complex time GF**

This is:



GC\* is supposed to be symmetric, in that GC\*(τ-β) = GC\*(τ). Let’s check,



So that works. Let’s get the Fourier transform,



So we have:



**Free phonon Green’s Functions**

Consider the lattice Hamiltonian,



we saw that we could reduce it to:



The causal GF, for instance, would be given by (using the CMT phase convention for the creation/annihilation operators):



where,



In the QM file we saw that we could reduce this to:



presuming the expectation is non-zero only when q´ = q and λ´ = λ, as is the case for a homogeneous isotropic medium at least. Apropos the complex time GF we want to consider, rather:



More particularly, we want the Fourier transform:



where:



It is straightfoward to show…



where,



Remember that boson number is not conserved and so there is no chemical potential. We can determine the complex time GF from,



which satisfies (for 0 < τ < β),



Now taking the FT we have:



So we have:



Again we observe that the poles of the GF are the excitations of the lattice. The advanced and retarded phonon Green’s functions are simpler (again, including the exponential convergence factors implicitly attached to θ’s when taking FT). So,



and taking the Fourier transform, adding in the implicit e-η(t-t´) that comes with the step function,



So we have:



and similarly, we’ll find:



And taking the Fourier transform will give us:



Now let’s take a look at the causal GF,



(I guess we presume n-qλ = nqλ, which would be true, at least Thermodynamically). And then taking the temporal transform, with aid of implicit convergence factor, we have,



And we can keep going,



We can say 2iΩqλη is just iη. And can neglect factor of η in numerator, and η2 in denominator. So,



But people (Mahan) don’t write it this way either. Now we’ll use the identity:



where P(1/z) means ‘take the principal value’ when doing integrals). And then we can say,



So this is our final formula,



If we do anti-causal GF, we’ll get:



and,



**Phonon G>**

Let’s consider the ‘greater’ GF.



We’ll say Ω-q = Ωq. I think it almost always is. And so n-qλ = nqλ.



So there.



**Phonon G<**

Let’s consider the ‘lesser’ GF.



We’ll say Ω-q = Ωq. I think it almost always is. And so n-qλ = nqλ.



So there.



Of course these expressions all follow from analytic continuation vis a vis the formulas in terms of the spectral function in the Formal Properties file.

**Spectral Function**

The spectral density is:



Note that,



which satisfies the general relation,

