**Formal Properties of Green’s Functions**

and going…

**Operator Green’s function**

Operator GF’s are sometimes employed. Their concept makes it convenient to put the Green’s functions in different bases. Consider a generic Green’s function,



where the bracket can stand for a commuator, or nothing or whatever. Then the operator Green’s function would be defined as:



If a GF in a particular basis is desired, say the momentum basis, then we just form,



**Differential equations obeyed by single particle Green’s functions**

Let A = ψ, and B = ψ†, and then we’ll investigate the differential equations obeyed by the complex time Green’s function. First, in real time, the time development of ψ is,



and so in complex time (t → -iτ), and updating to thermal averaging, we have:



Now suppose that we have a Hamiltonian with just a spin-dependent single particle potential:



Then, as we’ll recall we worked out in the QM folder/2nd quantization/Various bases file, we have, in real time:



It follows that the equation of motion for ψσH(τ) is (translating to complex time τ = it, and upgrading to K1 = H1 – μN, appropriate for the time development of thermal averaged GF’s):



And we can see that the equation for the GF is:



Recognizing the commutator gives us the δσσ´δ(x-x´) function, then we get,



Let’s step up to an interaction potential (spin – independent):



which in 2nd quantized notation is:



Then, from the QM folder/2nd quantization/Various bases file, we have, in real time,



where



Translating to complex time, and updating to K, appropriate for thermal averaging, we have:



So now consider the differential equation obeyed by GC\*(r,τ;r´,τ´). We find:



And we get:



where the interaction term is defined as:



and ρ is the density operator of course. The equations for the other guys: GC, GR, G>, etc., are basically the same as in the Quantum Mechanics folder, we just replace the state against which we’re taking an expectation with the thermal average. We could do a mean field approximation:



which would give us:



The differential equation obeyed by the operator GF’s will, in the mean field approximation (which is exact if no two-particle interaction) look like:



where H(r) = H1(r) + <H2(r)>. Now consider the distinct harmonic particles’ GF. We may imagine that, letting A = xα(**R**) and B = xβ(**R**), making the analogous set of calculations with these guys, we will get, say:



where,



and Kαβ is of course the ‘spring constant’, not the Kamiltonian (anyway, lattice has no μ). Might help to refer to the Quantum Mechanics/Distinct Particles/GF Formal Properties file.