**Quantum Perturbation Theory on Z**

Usually, we cannot calculate S, F, or L exactly since we’d need the exact energy levels, and usually particles are interacting which makes this a rather complicated thing to get. So there are of course perturbative methods of calculating these guys. And then we’ll discuss is a classical method called the *cluster expansion*.

**Perturbative expansion of eigenfunction/energies**

In the same spirit that we created a perturbative expansion of Z in a classical system, we can can create a perturbative expansion of f in the quantum case. The density matrix is given by:



and we can perturbatively calculate corrections to the many body wavefunction and the eigenenergies, to get:



where,



So to first order, we often make the approximation,



And



Additionally, the zeroth order approximation for interacting particles is the free situation. And if the perturbation doesn’t mix the particles, then we can just calculate corrections to the individual particle wavefunctions. So then we can proceed with the above, and come to the conclusion that:



is the many-body operator, and is the single-body operator. For instance, if calculating the density, then we’d have,



and in the single-body case is:



But calculating corrections to the wavefunctions and energies is usually a laborious process. Perturbative calculations of GF’s are easier. So we look for a way to get the density matrix, or partition function rather, from GF’s.

**Quantum cluster expansion**

So we want to evaluate:



where L = -kTlnΞ is the Landau free energy. We can use the GF method to evaluate this. We start with,



And then perform a perturbative expansion.



where the expectation is w/r to the non-interacting K of course. In evaluating the RHS, all diagrams are included, whether *connected or disconnected* since there is no other series to cancel the disconnected diagrams.



But it’s better to evaluate each different type of diagram only once. The way of doing this expansion is called the linked cluster expansion. The basic linked cluster theorem is that this series can be resumed into:



Where Un simply contains all the nth order *connected* diagrams – including topologically similar diagrams. Another way we can do say this – ridding ourselves of the topologically equivalent diagrams is:



so in this formulation we’d include nth order *topologically unique connected* diagrams. Finally, just to add one more highlighted equation, we can now say:



Usually an ∞ subset of U diagrams must be summed to get reasonable results.

**Example. Z for HO**

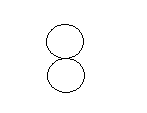
Let’s consider the usual cuartic HO.



For this situation we have no chemical potential, in which case the Helmholtz free energy is the same as the Landau free energy. What would be the leading correction to the (Helmholtz) free-energy? This would be:



The first order diagram would be:



which is:



where we used that identity in the Math Appendix for ‘Matsubara’ sums. And this will work out to:



So,



Now consider what we would’ve gotten from a direct evaluation of Z with the new excitation energies obtained earlier. We found in the self-energy file that:



and so this would give us:



That definitely doesn’t match. Should it? Probably not.