**Solid**

**Prologue**

Now we want to investigate how we may determine the functional form of these potentials introduced previously. We will consider for simplicity that our substance is homogeneous and obeys the scaling relations introduced in the previous lecture. Then our potentials satisfy,



It will suffice, then, to determine the heat capacity CV(T,V) and equation of state p(T,V), to construct the potentials [these basically give us S and p for the free energy F]. For instance, forming the differential of U(T,V) we have:



What are these derivatives? Well,



and,



So we have:



A similar procedure for the entropy would yield:



What are these derivatives? Well,



and,



So we have:



With U(T,V) and S(T,V), we can get F(T,V) = U(T,V) – TS(T,V) for instance, which is one of the thermodynamic potentials. So from this we can construct everything we want. And of course we can invert these equations to construct other potentials as desired.

**Solids**

Now let’s consider a solid. We typically have, in the high T regime, the following for the heat capacity, at some particular volume.



How CV might depend on volume we don’t necessarily know. So we’ll just leave it as is. For the equation of state, it is usually convenient to define two ancillary quantities, the coefficient of thermal expansion, α, and the isothermal compressibility, β (also called κT, and also 1/B, where B is the isothermal bulk modulus).



from which it follows that the differential of V(T,p) is given by:



For solids, these coefficients are approximately constant, and so we can integrate and obtain:



So,



So now let’s use these to get U. Well we have:



We’d like to integrate. But we do not necessarily know if CV depends on V or not and so cannot really integrate yet. But it turns out we can determine this information from the equation of state! For instance,



(remember that the variable subscripts aren’t saying that we’re holding those things constant per se´ - its more like they are telling us what the independent variables are). So we see that CV can only depend on T, given the equation of state! Integrating we get:



We could define U0 so as to eliminate the constant and so obtain:



Multiplying by N, and presuming CV = fk/2, we have:



Let V0 = υ when p0 = αT0/β, whatever that is. Then we could write this as:



The spring-like nature of the energy is clearly evinced, with 1/βυ serving as the spring constant. Now let’s consider the entropy. We have:



Integrating we get:



Again if we assume a constant (and usual) heat capacity we’ll have:



Multiplying through by N, defining a constant, and setting V0 = υ as before, we’ll have:



And we could continue as usual. Let’s now consider an interesting approach to getting the chemical potential of our solid. First we’ll get a formula for the entropy in terms of T and p.



Integrating we get:



Now V doesn’t change much with pressure. So we can say:



With S, the chemical potential can be obtained from the Gibbs-Duhem equation:



**More Elasticity Stuff**

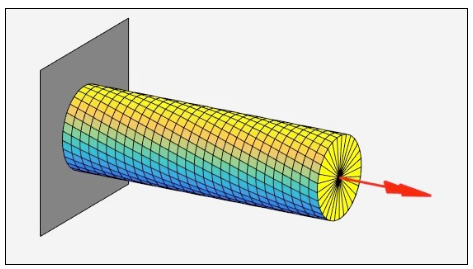
Say we have a pipe of length L and diameter d. If we apply a longitudinal strain, ΔL/L, then there will be a transverse strain as well, Δd/d. These are related via:



Makes sense that the diameter should get smaller if length gets longer right? Well, I guess there isn’t a conservation of volume thing *necessarily*, because sometimes ν can be negative. The Young’s and Shear moduli are related via ν.



Here’s another thing. If we apply a torsional torque to a bar, then it will twist.



The angle it will twist follows a stress-strain equation:

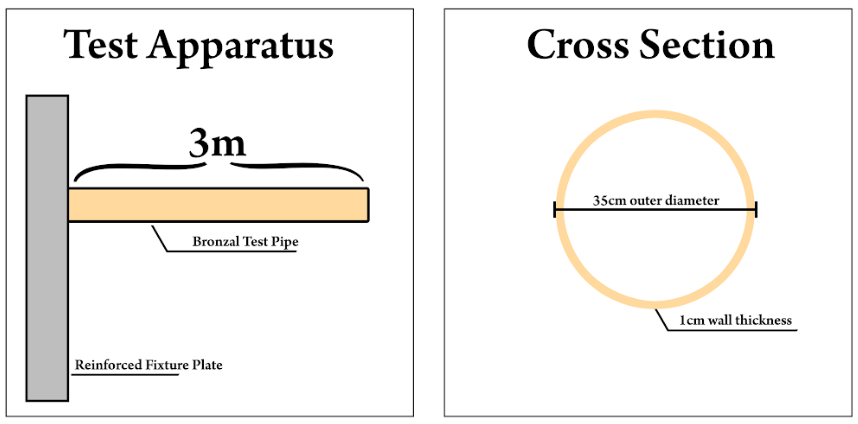


J is the polar moment of inertia that quantifies an object’s resistance to twisting. A pipe with inner and outer radii has the following J:



**Example**

A backyard scientist was working on a design for a novel heat exchanger, and was testing the properties of a coolant pipe he designed. The test pipe was cast in a bronzal alloy the scientist was investigating. The scientist tested the alloy, finding it had a modulus of elasticity of 113 GPa and a Poisson's ratio of 0.34. After casting, the pipe was polished inside and out, and was placed in a test apparatus as shown in Figure 1. One end of the pipe was fixed to a plate as shown in the above diagram, and the other end was subjected to a torque of 100,000 N-m. Assume that the fixture plate did not measurably deform during the test. Which of the following values most closely matches the twist the scientist observed in the test pipe?



So,



Filling in the values,



We get:

